**The accuracy of, and sensitivity to, tests for asymptotic dependence in spatial extreme rainfall models**

**Lee Fawcett, David Walshaw**

School of Mathematics, Statistics and Physics, Herschel Building, Newcastle University, Newcastle upon Tyne NE1 7RU, UK. E-mail: [David.Walshaw@ncl.ac.uk](mailto:%20David.Walshaw@ncl.ac.uk)

**Abstract**

Models for extreme rainfall, and resulting contour maps for quantities such as the 50-year return level, are greatly improved by spatial models which incorporate the dependence in extremes between locations. However, when considering extremal dependence between any pair of locations, we need to consider the fundamentally different classes of asymptotic independence and asymptotic dependence. In terms of the random variables X and Y representing the rainfall at two different locations, and at the same point in time, asymptotic independence and asymptotic dependence are characterized by lim[Pr(X>u|Y>u)] = 0 and lim[Pr(X>u|Y>u)] > 0 respectively, where the limit is taken as the threshold u increases to infinity.

Various tests have been developed to discriminate between the two classes, in particular based on the so-called tail-dependence index ɳ , and tests for its value, noting that asymptotic dependence is represented by the case ɳ = 1. These tests are often based on the so-called Hill estimator, and modifications thereof. However, the discrimination task is notoriously difficult, with the reliability of tests being suspect at best.

In this paper we systematically investigate the performance of preferred estimators for the tail-dependence index ɳ , using a simulation study in which the extremal dependence status of the models which give rise to artificial data is known in each case.

We illustrate and quantify the rather poor performance of the preferred estimators which are routinely used in the field. We analyse the consequences, in terms of the sensitivity of findings based on the resulting misleading conclusions, when these are used in typical fashion to estimate things like simultaneous extremes at multiple locations.

We show that the large biases incurred, in situations where the test misclassifies the dependence class, could greatly underestimate the probabilities of catastrophes, and we consider some mitigation strategies.